Bounding the Metric Dimension of a Graph with Its Local Metric Dimension and Multiset Dimension

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The concept of metric dimension of graphs was introduced to uniquely identify the location of a vertex in a nontrivial connected graph G. An ordered set of vertices is utilised as a landmark to identify a vertex v in G by determining the vector of distances between v and all vertices in the ordered set. Formally, for an ordered set $W = \{w_1, w_2, \ldots, w_k\}$ of vertices in G, the representation of a vertex v of G with respect to W is the k-vector $r(v|W) = (d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$. An ordered set W resolves G if $r(u|W) \neq r(v|W)$ for every pair of distinct vertices u, v in G. A resolving set of minimum cardinality is a basis of G, and its cardinality is the metric dimension of G, $dim(G)$.

In the case where only adjacent vertices should be uniquely identified, then an ordered set W is said to locally resolve G if every pair of adjacent vertices in G have distinct representations. A minimum local resolving set is a local basis, and its cardinality is the local metric dimension of G, $lmd(G)$.

On the other hand, we could release the ordered condition of a landmark that will result in a vertex representation which is a multiset, instead of a vector. For W a subset of vertices in G, the representation multiset of a vertex v with respect to W, $r_m(v|W)$, is a multiset of distances between v and the vertices in W. If $r_m(u|W) \neq r_m(v|W)$ for every pair of distinct vertices u and v, then W is called an unordered resolving set of G. If G has an unordered resolving set, then the cardinality of a smallest unordered resolving set is called the multiset dimension of G, denoted by $md(G)$; otherwise we say that G has infinite multiset dimension, $md(G) = \infty$.

It is obvious that every unordered resolving set is a resolving set, and every resolving set is a local resolving set; and so

$$lmd(G) \leq dim(G) \leq md(G).$$

In this talk, I will present a short historical account, recent results, and open problems for metric dimension, local metric dimension, and multiset dimension of graphs. Comparison among the three kinds of dimensions will also be discussed.