Hamiltonian cycles in tough $P_2 \cup P_3$-free graphs

Songling Shan, Illinois State University*

Let $t > 0$ be a real number and G be a graph. We say G is t-tough if for every cutset S of G, the ratio of $|S|$ to the number of components of $G - S$ is at least t. Determining toughness is an NP-hard problem for arbitrary graphs. The Toughness Conjecture of Chvátal, stating that there exists a constant t_0 such that every t_0-tough graph with at least three vertices is hamiltonian, is still open in general. A graph is called $P_2 \cup P_3$-free if it does not contain any induced subgraph isomorphic to $P_2 \cup P_3$, the union of two vertex-disjoint paths of order 2 and 3, respectively. We show that every 15-tough $P_2 \cup P_3$-free graph with at least three vertices is hamiltonian.

Keywords: Toughness; Hamiltonian cycle; $P_2 \cup P_3$-free graph