On Hb-graphs and their application to hypergraph e-adjacency tensor

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In Ouvrard et al. (2017) we introduced for hypergraphs the concepts of k-adjacency - k given vertices are in a hyperedge - and e-adjacency - given an hyperedge, its vertices are e-adjacent - extending the classical concept of adjacency which corresponds to 2-adjacency. We also introduced $\overline{k}$-adjacency which is the maximal k-adjacency that can be found in a given hypergraph.

For k-uniform hypergraphs, $\overline{k}$-adjacency corresponds to k-adjacency, and is equivalent to e-adjacency; work on k-adjacency tensor representations exist (Drineas and Lim (2005), Cooper and Dutle (2012), Hu (2013)). For general hypergraphs, $\overline{k}$-adjacency is no more equivalent to e-adjacency; existing e-adjacency tensors (Banerjee et al. (2017), Ouvrard et al. (2017)) show limitations either in their interpretability or in spectral applications. To overcome these difficulties Ouvrard et al. (2017) introduced a hypergraph uniformisation process that allows to transform a hypergraph into a weighted uniform hypergraph, and a polynomial homogeneisation process that combined enable the construction of a general hypergraph e-adjacency tensor. This uniformisation process imposes a decomposition pf the original hypergraph into layers of uniform hypergraphs of increasing hyperedge cardinality. The hypergraph uniformisation process consists in an iterative two-phase step of inflation and merging applied to the uniform hypergraphs contained in the successive layers: the uniform hypergraph of the lower layer is inflated with a new vertex and merged with the uniform hypergraph - eventually empty - of the next layer.

Simplifying this process requires to handle duplication of vertices. Multisets - also known as bags - allow to do it (Syropoulos (2000)) and have strong applications in databases (Lamperti et al.(2000); Singh et al. (2007)). In this talk, based on Ouvrard et al. (2018), we introduce the concept of hyper-bag-graphs (hb-graphs for short), an extension of hypergraphs to families of multisets. Hb-graphs have already shown their efficiency for retrieving the importance of vertices and hb-edges in co-occurence networks using exchange-based diffusion (Ouvrard et al. (2018)): the information stored in hb-graphs is higher than that stored in hypergraphs, allowing optimal knowledge discovery.

Using a hb-graph m-uniformisation process, we construct three hb-graph e-adjacency tensors. Hypergraph being special case of hb-graph, these tensors allow to propose two new e-adjacency tensors for hypergraphs, the third one being linked to the one proposed in Ouvrard et al. (2017). We conclude our talk by giving some first results on hypergraph spectral analysis of the built tensors and a comparison with the existing tensors.

Keywords: hb-graph, multiset, hypergraph e-adjacency, e-adjacency tensor