Graph decompositions of 7 vertices and 8 edges

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We will prove that 13 out of the 16 non-isomorphic connected two cycled graphs with seven vertices and eight edges decompose the complete graph K_n whenever the necessary conditions are satisfied. For the 3 remaining graphs, we will present partial results.

If G is a graph with n vertices, we say that the complete graph K_n has a G-decomposition if there are subgraphs $G_1, G_2, \cdots, G_s$ of K_n, all isomorphic to G, such that each edge of K_n belongs to exactly one G_i with some index $1 \leq i \leq s$. It has been proved that if a graph G admits certain types of labelings, then it forms a G-decomposition of K_n. For our case, we used σ-labelings of multiple copies of G to prove the existence of G-decompositions for bicyclic graphs on 7 vertices and 8 edges.

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