Counting Vertices in Tessellations of the Hyperbolic Plane

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Let $T(d, f)$ be some planar, d-regular graph such that every face has f sides. For every face of $T(d, f)$ to be a regular polygon, it tessellates a sphere, the plane, or the hyperbolic plane. We focus on counting vertices in tessellations of the hyperbolic plane. When $T(d, f)$ induces a tessellation of the hyperbolic plane, we can draw the graph starting with some vertex v and go out in rings of faces away from v. We offer a proof for a closed form solution for the number of vertices in the n-th ring from v when $d \geq 3$, $f \geq 4$, and $T(d, f)$ induces a tessellation of the hyperbolic plane, and we offer a proof for a different closed form solution when $d \geq 7$, $f = 3$, and $T(d, f)$ induces a tessellation of the hyperbolic plane.

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