Linear Operators on Graphs: Genus Preservers

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Let m and n be positive integers, and let $R = (r_1, \ldots, r_m)$ and $S = (s_1, \ldots, s_n)$ be nonnegative integral vectors. Let $A(R, S)$ be the set of all $m \times n$ $(0, 1)$-matrices with row sum vector R and column vector S. Let R and S be nonincreasing, and let $F(R, S)$ be the $m \times n$ $(0, 1)$-matrix where for each i, the i^{th} row of $F(R, S)$ consists of r_i 1's followed by $n - r_i$ 0's. Let A be a $(0, 1)$-matrix in $A(R, S)$. The discrepancy of A, $disc(A)$, is the number of positions in which $F(R, S)$ has a 1 and A has a 0. In this paper we investigate linear operators mapping $m \times n$ matrices over the binary Boolean semiring to itself that preserve sets related to the discrepancy. In particular we show that bijective linear preservers of Ferrers matrices are either the identity mapping or, when $m = n$, the transpose mapping.

Key words and phrases: Ferrers matrix, nested bipartite network, discrepancy, linear preserver, strong linear preserver.