A Study of Time Varying Copula Approach to Oil and Stock Market

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Dedication

To my parents and those who supported me through my graduate study.

Abstract

In this project we analyze stock data and consider the multivariate dependence between OPEC oil prices and SP500 and NASDAQ stock market prices in United States. We use time-varying copulas to model the dependence structure. Our analysis indicates that there is positive dependence between oil prices and stock markets data in United States, particularly during a financial crisis. We also find out that among copula models under consideration, the student-t copula is the best candidate to describe the dependence structure for daily data, while for weekly data the Clayton copula is the best.

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CHAPTER 1
Introduction

In risk management, copulas are used to perform test and studies for financial risk where extreme downside events may occur. Modeling dependence with copula functions is widely used in various dependence studies and financial risk assessment studies and risk analysis.

Oil price has been a crucial factor in the globe economy, therefore many economic and financial studies focus on oil price impact on stock markets. However those studies contain mix results as Apergies and Miller (2009) found stock market prices do not react to volatility of the oil market price. Then Dhaoui and Khraief (2014) found that oil market prices and stock market price are negatively correlated in US and EU countries stock market volatility, while Park and Ratti (2008) discovered that oil stock market prices are significantly positively correlated in US and EU stock markets. The conclusions are mixed as they employed different methods in their studies.

There are various copula models available to study and analyze, but selecting better fitting models to use is crucial. In our study, we model the dependence between oil market prices and US stock market prices as suggested by Alouia, Hammoudehb, Nguyenc (2013), and Chen and Fan (2006). By using Cramér–von Mises method suggested by Genest (2009) we test and compare the performance of different copulas to determine which copula model is better in the oil and stock market studies.

CHAPTER 2
Empirical Model

In this project, we use a time-varying copula approach to study the dependence structure between OPEC oil price and US stock market returns: SP500 and NASDAQ. For my study, we have chosen to use the data of a ten-year period between 2005 to 2015. I begin by using the ARMA-GARCH model to the data to extract the standardized residuals, then use the standardized residuals to estimate several copula models.

2.1 ARMA-GARCH Model

The ARMA-GARCH model is one of the commonly used in financial time series to calculate time varying volatility.

Given a time series y_t an ARMA(m,n)-GARCH(1,1) model is defined by

$$\begin{aligned} y_t &= \mu + \sum_{i=1}^{m} a_i y_{t-i} + \sum_{j=1}^{n} b_j \epsilon_{t-j} + \epsilon_t, \\ \epsilon_t &= \sigma_t z_t, \\ \sigma_t^2 &= \omega_0 + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \end{aligned} \tag{1}$$

where z_t is a sequence of i.i.d (independent and identically distributed) random variables with mean zero and variance one. μ is the conditional mean, and σ_t^2 is the conditional variance of return series at time t. The a_i are autoregressive coefficients, b_j are moving average coefficients, and $\omega_0 > 0, \alpha \geq 0, \beta \geq 0$ are unknown.

2.2 Copula

Copula was first introduced by Sklar in 1959 and became very popular in financial analysis in the past decade. One can use copula models to measure dependence structure of financial data for better understanding and managing portfolios.

Definition: Copula $C: [0,1]^d \rightarrow [0,1]$ is a d-dimensional copula if C is a joint cumulative distribution function of a d-dimensional random vector on the unit cube $[0,1]^d$ with uniform marginals (Nelsen, 2006).

2.3 Sklar's theorem

Let $X_1, \dots X_d$, be random variables with marginal distribution functions $F_i(x) = P[X_i \leq x]$ and H be joint distribution

$$H(x_1, \dots, x_d) = P[X_1 \leq x_1 \dots X_d \leq x_d]. \tag{2}$$

Then there is a copula C: $[0,1]^d \rightarrow [0,1]$, such that

$$H(x_1, \dots, x_d) = C[F(x_1), \dots . F(x_d)]. \tag{3}$$

Copula functions are efficient to create distributions to model correlated multivariate data. We can construct a multivariate joint distributions by first specifying marginal univariate distributions then choosing a copula to examine the correlation structure between variables. A copula can also characterize the tail dependence coefficients to help measure the comonotonicity of random variables.

Let X and Y be random variables with marginal distribution functions F and G. Then the coefficients of lower and upper tail dependence λ_L and λ_U are defined as

$$\lambda_L = \lim_{t \to 0^+} \Pr[Y \leq G^{-1}(t) | X \leq F^{-1}(t)],$$

$$\lambda_U = \lim_{t \to 1^-} \Pr[Y \leq G^{-1}(t) | X \leq F^{-1}(t)]. \tag{4}$$

If λ_U and λ_L are equal to each other then there is a symmetric tail dependence between the two assets, otherwise they are asymmetric. The tail dependence coefficients also provide a way to compare different copulas in that if there are two copulas, the one with higher tail dependence coefficient is more concordant than the one with lower tail dependence coefficient.

The copula I am considering to use in my tests are copulas from the Elliptical copula family: Gaussian (normal), Student-t, and Archimedean copula family: Gumbel, Clayton, Frank and Joe. Those copula functions are briefly explained below.

2.4 Copula Models

2.4.1 Bivariate Gaussian (Normal) Copula

The Gaussian copula is a symmetric copula which exhibits no tail dependence.

$$C(u, v) = \phi_\theta(\phi^{-1}(u), \phi^{-1}(v)) \tag{5}$$

$$= \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp\left(-\frac{s^2 - 2\theta st + t^2}{2(1-\theta^2)}\right) dsdt,$$

where ϕ_θ is the joint distribution of two standard normal random variables with correlation coefficient $\theta \in (-1,1)$, and ϕ is the cumulative distribution of the standard normal.

2.4.2 Bivariate Student-t Copula

The Student-t copula is used to capture extreme dependence between variables.

$$C(u,v) = \int_{-\infty}^{t_{df}^{-1}(u)} \int_{-\infty}^{t_{df}^{-1}(v)} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp\left(1 + \frac{s^2 - 2\theta st + t^2}{df(1-\theta^2)}\right) dsdt, \tag{6}$$

where $t_{df}^{-1}(u)$ and $t_{df}^{-1}(v)$ denotes the inverse of the CDF of the standard univariate student-t distribution with df degrees of freedom.

2.4.3 Gumbel Copula

The Gumbel copula is an asymmetric copula exhibiting greater dependence in the positive tail than the negative.

$$C(u,v) = \exp\left\{-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{1/\theta}\right\}, \text{where } 0 < \theta < \infty. \tag{7}$$

2.4.4 Clayton Copula

The Clayton Copula is an asymmetric copula exhibiting greater dependence in the negative tail than the positive.

$$C(u,v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{\frac{1}{\theta}}, \text{ where } 0 < \theta < \infty. \tag{8}$$

2.4.5 Frank Copula

$$C(u,v) = -\frac{1}{\theta}\ln\left(1 + \frac{\left(e^{-\theta u} - 1\right)\left(e^{-\theta v} - 1\right)}{e^{-\theta} - 1}\right), \text{where} -\infty < \theta < \infty. \qquad (9)$$

2.4.6 Joe Copula

$$C(u,v) = 1 - \left[(1-u)^{\theta} + (1-v)^{\theta} - (1-u)^{\theta}(1-v)^{\theta}\right]^{\frac{1}{\theta}}, \text{where } 1 \le \theta < \infty. \qquad (10)$$

2.5 Choosing Suitable Copula Models

2.5.1 Canonical Maximum Likelihood (CML)

We used Canonical Maximum Likelihood to estimate parameter $\hat{\theta}_n$, which is consistent.

$$\hat{\theta}_n = \arg_{\theta} \max \sum_{i=1}^{n} \ln c\left(\hat{F}_X(x_i), \hat{F}_y(y_i); \theta\right), \qquad (11)$$

where $\hat{F}_X(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \le x)$, $\hat{F}_Y(y) = \frac{1}{n}\sum_{i=1}^{n} I(Y_i \le y)$,

n is the number of observations, and c is the density function of a Copula C.

2.5.2 Cramer Von Mises Statistic

To compare the copula models, we use the goodness-of-fit test which is based on a comparison of the distance between the estimated and empirical copula by using the Cramer Von Mises statistic method. Set d = 2. Let $(X_1, Y_1) \ldots (X_n, Y_n)$ be a sequence of n i.i.d. random vectors with copula C and

denote $X_{(i)}$ as the i-th smallest observation of $X_1, \dots, X_n$ and $Y_{(i)}$ as the i-th smallest observation of $Y_1, \dots, Y_n$. Then the empirical distribution based on $(X_1, Y_1) \dots (X_n, Y_n)$ is defined as

$$H_n(x, y) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \le x, Y_i \le y).$$

Then it follows from (3) that the empirical estimator of C is given by

$$C_n(u, v) = H_n(X_{(nu)}, Y_{(nv)}).$$

The Cramer Von Mises statistic is given by

$$S_n = n\int \left(C_n(u, v) - C_{\hat{\theta}_n}(u, v)\right)^2 dudv. \tag{12}$$

To get the P-value of the Cramer Von Mises statistic, we adapt a bootstrap method described in Kojadinovic and Yan (2011). Choose a large integer N, repeat the following steps for every $k \in \{1 \dots N\}$

1) Generate a random sample $(X_{1k}, Y_{1k}) \dots (X_{nk}, Y_{nk})$ from copula $C_{\hat{\theta}_n}$;
2) Let C_{nk} and θ_{nk} stand for the C_n and θ_n base on $(X_{1k}, Y_{1k}) \dots (X_{nk}, Y_{nk})$;
3) Compute

$$S_{nk} = n\int \left(C_{nk}(u, v) - C_{\hat{\theta}_{nk}}(u, v)\right)^2 dudv\,;$$

4) An approximate P-value for the test is given by

$$\frac{1}{N}\sum_{k=1}^{N} I(S_{nk} \ge S_n). \tag{13}$$

CHAPTER 3
Data and Results

I used the daily closing price data for the OPEC Oil index, and the SP500 and NASDAQ daily close over the period from January 1, 2005 to January 1, 2015, resulting a total of 2579 observations. Note that there were two financial crisis occurred in the ten-year period. The Oil data is obtained from U.S. Energy Information Administration database. The SP500 and NASDAQ data is obtained from the Yahoo Finance database. Both data were downloaded with the R package Quandl which can directly help you download historical data from various databases. I used log-returns of the data for my analysis. The function for the log return is $\mathrm{r_t = \ln(P_t/P_{t-1})}$ where $\mathrm{P_t}$ is the index or price at time t. The time varying log returns are plotted in Figure 1. From Figure 1, the daily returns are fairly stable before the Financial crisis (prior to Fall 2007) and exhibit higher variability afterwards until Spring 2012 and start become fairly stable again.

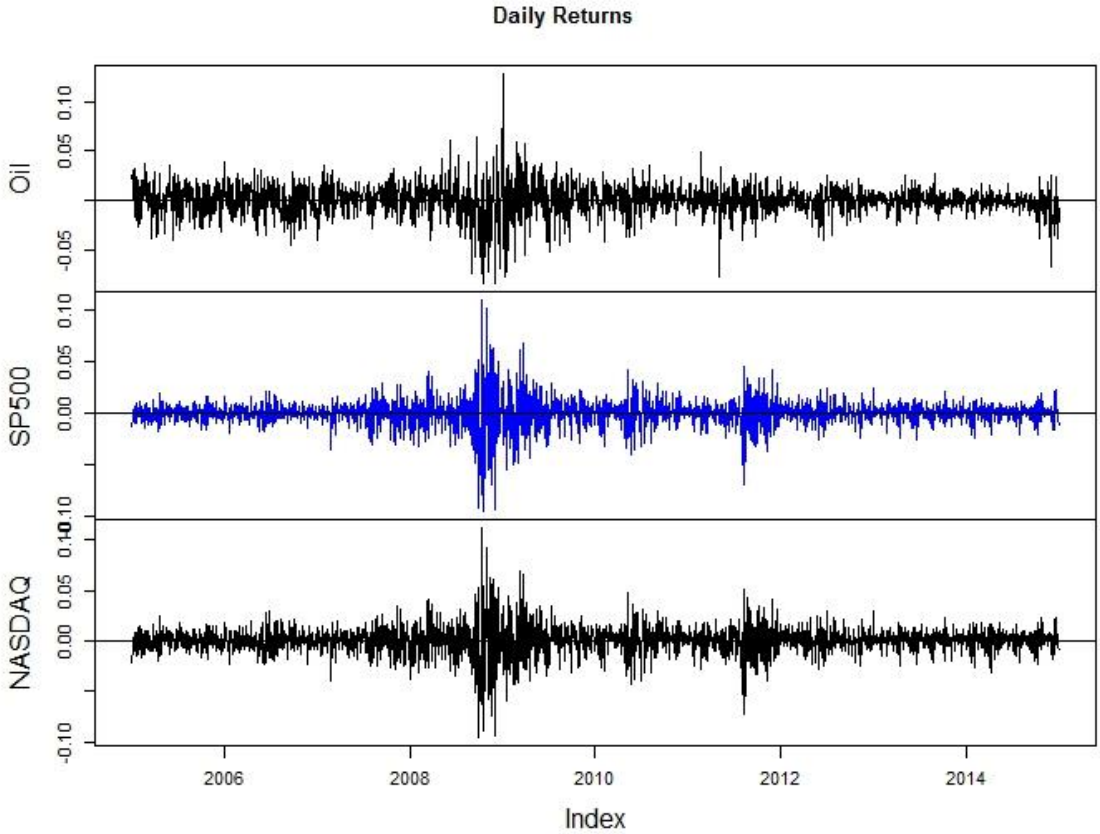

Figure 1: Volatility of Oil Market price and SP500 and NASADAQ daily

Table 1 presents the descriptive statistics for the returns in our study. The averages for all three market returns are relatively positive and standard deviation is relatively similar in all the market returns too. The skewness coefficients are all negative and excess kurtosis ranges from 4.06 to 9.21. Those results show strong rejection by the Jarque-Bera's normality test, therefore those data have higher negative or positive skewness.

Daily	mean	sd	min	max	range	skew	Ex. kurtosis
Oil	0.00015	0.04	-0.25	0.19	0.45	-0.77	4.06
SP500	0.000214	0.03	-0.2	0.11	0.31	-0.97	9.21
NASDAQ	0.000314	0.03	-0.17	0.1	0.27	-0.8	5.08

Table 1: Descriptive statistics of Daily returns on Oil Market price and SP500 and NASADAQ

To find the dependence structure between the oil market price and each of the stock markets, we first need to filter the returns using an appropriate ARMA(m,n)-GARCH(1,1) process. We used tools developed by Yohan Chalabi in R (fgarch) to help in selecting the best ARMA-GARCH model. Our analysis resulted in ARMA(1,1)-GARCH(1,1) for all three returns. The objective of using ARMA-GARCH to filter these returns is to "approximate Independent and identically distributed (i.i.d) residuals while controlling for effects of conditional heteroscedasticity" (Aloui 2013). Then we estimate the marginal distributions using filtered returns and apply the CML method to determine the unknown parameter θ for our copula models: Gaussian, Student-t, Gumbel, Clayton, Frank and Joe copula models.

Table 2 shows the estimated values of dependence parameters for pairs of oil price and SP500 and pairs of oil price and NASDAQ for each copula model in use. The result shows that the dependence parameters are all positive and indicate that the prices of oil markets and US stock markets have positive correlation.

Daily	Gaussian	Student-t	Gumbel	Clayton	Frank	Joe
SP500	0.149 (0.019)	0.232 (0.049) d=3.815 (0.362)	1.120 (0.016)	0.200 (0.025)	0.898 (0.125)	1.144 (0.023)
NASDAQ	0.135 (0.019)	0.134 (0.023) d=3.805 (0.806)	1.108 (0.016)	0.186 (0.024)	1.814 (0.126)	1.128 (0.023)

Table 2: Estimated copula dependence parameters for Gaussian, Student-t, Gumbel, Clayton, Frank and Joe copula models on daily data. Standard error are in parentheses

Table 3 is the result of the Cramér Von Mises method and goodness of fit test. The hypothesis is that the estimated copula is a good fit for the data and rejected for p-values less than 0.05. The result shows that the Student-t copula yields the smallest distance between fitted and empirical copula and highest p-value, also all other copula hypotheses are rejected at significance level of 0.05. Table 4 also suggests that the model exhibits more lower tail dependence than its upper tail. Therefore it suggests that the student-t copula is the best candidate among the copula models we tested to use to construct dependence structures.

Daily	Gaussian	Student-t	Gumbel	Clayton	Frank	Joe
SP500	0.052 (0.01)	**0.022** **(0.3)**	0.074 (0.00)	0.052 (0.01)	0.059 (0.00)	0.15 (0.00)
NASDAQ	0.059 (0.00)	**0.024** **(0.2)**	0.083 (0.00)	0.047 (0.03)	0.067 (0.00)	0.15 (0.00)

Table 3:Distance between the empirical and estimated copula using Cramér–von Mises method. P-values are in parenthesis and bold numbers indicated the lowest distances among the test copula models.

	Lower	Upper
SP500	0.143	0
NASDAQ	0.132	0

Table 4:Tail dependence coefficient for daily data

We also apply the same method to studies of weekly data. Aroui and Nguyen (2010) suggest that weekly data may be more sensitive to capture the correlation and dependence of oil and stock market. Figure 2 shows the returns of weekly data, and gives an easier comparison between the returns. It shows larger volatility ranges than daily returns.

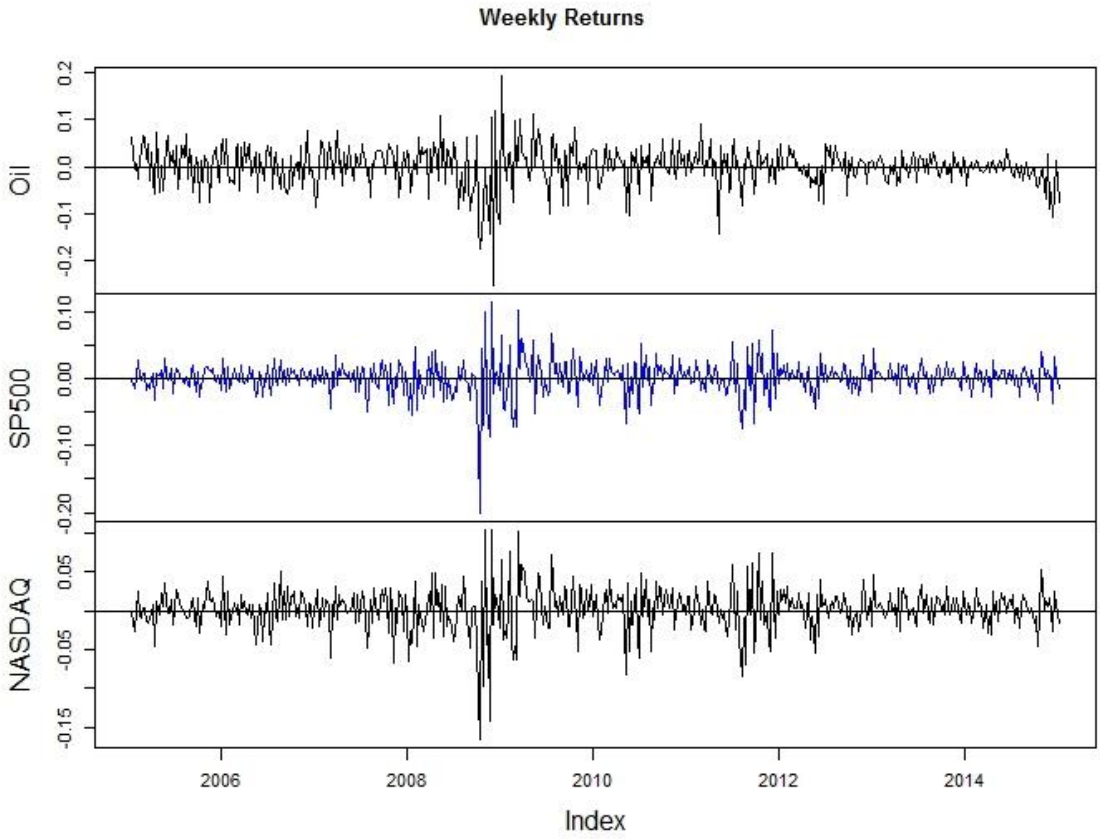

Figure 2: Volatility of Weekly returns of Oil Market price and SP500 and NASADAQ daily

The descriptive statistics of weekly returns in Table 5 show relatively the same results as daily returns. Table 6 contains the dependence parameters estimated from weekly returns which are relatively close to our daily returns for all copula models, but Table 7 which shows the goodness of fit test suggests that Clayton is a better fit than student-t distribution as it has the shortest distance between empirical and estimated copulas. However, Clayton also suggests greater dependence in the lower tail than the upper tail which is consistent with the results from daily return data with the student-t distribution. Also from Table

8 we can find that daily returns data have more fat tailed distribution than weekly return data. Therefore it is more effective to discover the dependence with higher frequency data so daily data is better to observe the dependence.

Weekly	mean	sd	min	max	range	skew	kurtosis
Oil	0.000535	0.04	-0.25	0.19	0.45	-0.77	4.06
SP500	0.001058	0.03	-0.2	0.11	0.31	-0.97	9.21
NASDAQ	0.001571	0.03	-0.17	0.1	0.27	-0.8	5.08

Table 5: Descriptive statistics of Daily returns on Oil Market price and SP500 and NASADAQ

Weekly	Gaussian	Student-t	Gumbel	Clayton	Frank	Joe
SP500	0.282 (0.038)	0.232 (0.049) df=3.557 (0.769)	1.217 (0.040)	0.349 (0.058)	1.474 (0.277)	1.257 (0.061)
NASDAQ	0.266 (0.039)	0.226 (0.049) df=3.704 (0.806)	1.196 (0.039)	0.342 (0.058)	1.401 (0.277)	1.215 (0.059)

Table 6: Estimated copula dependence parameters for Gaussian, Student-t Gumbel, Clayton, Frank and Joe copula models on weekly data. Standard error are in parentheses.

Weekly	Gaussian	Student-t	Gumbel	Clayton	Frank	Joe
SP500	0.062 (0.00)	0.053 (0.01)	0.070 (0.00)	**0.046** **(0.07)**	0.064 (0.00)	0.11 (0.00)
NASDAQ	0.056 (0.00)	0.042 (0.02)	0.076 (0.00)	**0.032** **(0.10)**	0.059 (0.00)	0.13 (0.00)

Table 7:Distance between the empirical and estimated copula using Cramér–von Mises method. P-value are in parenthesis and Bold number indicated the lowest distances amount the test copula models.

	Lower	Upper
SP500	0.103	0
NASDAQ	0.097	0

Table 8:Tail dependence coefficient for weekly data

CHAPTER 4
Conclusions

In this project, we apply time varying copula model approach to model the dependence between oil market price and US stock markets and compare the effectiveness between copula models by goodness of fit test, also we look at difference between frequencies of data's impact using same method of analysis. Overall, our results reveal positive dependence between oil and US stock markets. However, different copula models are selected to better fit and analyze the data between daily and weekly returns.

Reference

Aloui, R., Hammoudeh, S., & Nguyen, D. K. 2013. A time-varying copula approach to oil and stock market dependence: The case of transition economies. Energy Economics, 39, 4, 208-221.

Apergis, N., Miller, S.M., 2009. Do structural oil-market shocks affect stock prices? Energy Economics 31(4), 569-575.

Arnold, T. B., & Emerson, J. W. 2011. Nonparametric Goodness-of-Fit Tests for Discrete Null Distributions. R Journal, 3(2), 34

Chen, X., Fan, Y., 2006. Estimation of copula-based semiparametric time series models. Journal of Econometrics 130, 307-335.

Dhaoui, Abderrazak & Khraief, Naceur, 2014. "Empirical linkage between oil price and stock market returns and volatility: Evidence from international developed markets," Economics Discussion Papers 2014-12, Kiel Institute for the World Economy (IfW).

Fermanian, Jean-David 2013. "An overview of the goodness-of-fit test problem for copulas", in "Copulae in Mathematical and Quantitative Finance", P. Jaworski, F. Durant and W. Härdle (ed.), Springer (2013).

Genest, C., Kojadinovic, I., Nešlehová, J., & Yan, J. 2011. A goodness-of-fit test for bivariate extreme-value copulas. Bernoulli, 17(1), 253-275.

Genest, C., Rémillard, B., Beaudoin, D., 2009. Goodness-of-fittests for copulas: a review and a power study. Insurance: Mathematics and Economics 44, 199-213.

Nelsen, R. B. 2006. An Introduction to Copulas, Second Edition. New York, NY 10013, USA: Springer Science+Business Media Inc. ISBN 978-1-4419-2109-3.

Park, J., Ratti, R.A., 2008. Oil price shocks and stock markets in the U.S. and 13 European countries. Energy Economics 30, 2587-2608.

Sklar, A. 1959. Fonctions de répartition à n dimensions et leurs marges, Publ. Inst. Statist. Univ. Paris 8: 229–231

Wurtz, D., Y. Chalabi, and L. Luksan, 2006. Parameter Estimation of AR MA Mod-els with GARCH/APARCH Errors An R and SPlus Software Implementation. J.Statis. Soft..

R Code:

```
library(ggplot2)
library(copula)
library(Quandl)
library(rugarch)
library(fGarch)
library(psych)
library(rmgarch)
library(MASS)
library(PerformanceAnalytics)
library(quantmod)
library(mvtnorm)
library(mnormt) # needed for dmt
library(sn)
library(MTS)
```

Data Input For Daily Data

```
Start <- "2005-01-01"
End <- "2015-01-01"

Oil <- Quandl("OPEC/ORB", start_date= Start, end_date= End)
SP500 <- Quandl("YAHOO/INDEX_GSPC", start_date= Start, end_date=
End)[,c("Date","Close")]
NASDAQ <- Quandl("NASDAQOMX/COMP", start_date= Start, end_date=
End)[,c("Trade Date","Index Value")]
```

Data Input For Weekly Data

```
Start <- "2005-01-01"
End <- "2015-01-01"

Oil <- Quandl("OPEC/ORB", start_date= Start, end_date= End, collapse =
"weekly")
SP500 <- Quandl("YAHOO/INDEX_GSPC", start_date= Start, end_date= End,
collapse = "weekly")[,c("Date","Close")]
NASDAQ <- Quandl("NASDAQOMX/COMP", start_date= Start, end_date= End,
collapse = "weekly")[,c("Trade Date","Index Value")]
```

Code for Data Process

```
Oil.q <- Oil
SP500.q <- SP500
NASDAQ.q <- NASDAQ

Oil.t = as.ts(Oil.q)
SP500.t = as.ts(SP500.q)
NASDAQ.t = as.ts(NASDAQ.q)

names(Oil.q)[2] <- "Oil"
names(SP500.q)[2] <- "SP500"
names(NASDAQ.q)[1] <- "Date"
names(NASDAQ.q)[2] <- "NASDAQ"
rownames(Oil.q) <-Oil.q[,1]
Oil.q[,1] <- NULL
rownames(SP500.q) <-SP500.q[,1]
SP500.q[,1] <- NULL
rownames(NASDAQ.q) <- NASDAQ.q[,1]
NASDAQ.q[,1] <- NULL

Oil.ret = CalculateReturns(Oil.q, method="log")
SP500.ret = CalculateReturns(SP500.q, method="log")
NASDAQ.ret = CalculateReturns(NASDAQ.q, method = "log")


merge.data1 <- merge(Oil.ret, SP500.ret, by=0, all=TRUE)
rownames(merge.data1) <- merge.data1[,1]
merge.data1[,1] <- NULL
merge.data.ret <- merge(merge.data1, NASDAQ.ret, by=0, all=TRUE)
rownames(merge.data.ret) <- merge.data.ret[,1]
merge.data.ret[,1] <- NULL
merge.data.ret[is.na(merge.data.ret)] <- 0

Oil.ret <- merge.data[,1]
SP500.ret <- merge.data[,2]
NASDAQ.ret <- merge.data[,3]


Oil.SP500.ret = cbind(merge.data.ret[,1], merge.data.ret[,2])
Oil.NASDAQ.ret = cbind(merge.data.ret[,1], merge.data.ret[,3])

Oil.ret.t = as.ts(Oil.ret)
SP500.ret.t = as.ts(SP500.ret)
```

```
NASDAQ.ret.t = as.ts(NASDAQ.ret)

Oil.SP500.ret.t = as.ts(Oil.SP500.ret)
Oil.NASDAQ.ret.t = as.ts(Oil.NASDAQ.ret)
merge.data.ret.t = as.ts(merge.data.ret)
Oil.mu <- mean(Oil.ret)
Oil.sd <- sd(Oil.ret)
SP500.mu <- mean(SP500.ret)
SP500.sd <- sd(SP500.ret)
NASDAQ.mu <- mean(NASDAQ.ret)
NASDAQ.sd <- sd(NASDAQ.ret)
```

```
my.panel <- function(...) {
  lines(...)
  abline(h=0)
}

plot.zoo(merge.data.ret, main="Weekly Returns", panel=my.panel, col=c("black",
"blue"))


merge.data1 <- cbind(Oil.ret.t,SP500.ret.t, NASDAQ.ret.t)

describe(merge.data)
pairs.panels(merge.data1)


plot(coredata(Oil.ret), coredata(SP500.ret), main="Empirical Bivariate Distribution
of Returns", ylab="SP500", xlab="Oil", col="blue")
abline(h=mean(SP500.ret), v=mean(Oil.ret))

plot(coredata(Oil.ret), coredata(NASDAQ.ret), main="Empirical Bivariate
Distribution of Returns", ylab="NASDAQ", xlab="Oil", col="blue")
abline(h=mean(NASDAQ.ret), v=mean(Oil.ret))

plot(coredata(Oil.ret), coredata(SP500.ret),
    main="Empirical Bivariate Distribution of Returns",
    ylab="SP500", xlab="Oil", col="blue")
abline(h=mean(SP500.ret), v=mean(Oil.ret))

par(mfrow=c(1,3))
qqnorm(coredata(Oil.ret), main="Oil", ylab="Oil quantiles")
qqnorm(coredata(SP500.ret), main="SP500", ylab="SP500 quantiles")
```

```
qqnorm(coredata(NASDAQ.ret), main="NASDAQ", ylab="NASDAQ quantiles")
par(mfrow=c(1,1))

#######################################
# Pseudo Code for ARMA(1,1)-GARCH(1,1)   #
#######################################

fit1=garchFit(formula = ~ arma(1,1)+garch(1, 1),data=dat[,1],cond.dist ="std")
fit2=garchFit(formula = ~ arma(1,1)+garch(1, 1),data=dat[,2],cond.dist ="std")
fit3=garchFit(formula = ~ arma(1,1)+garch(1, 1),data=dat[,3],cond.dist ="std")
m_res <- apply(dat_res, 2, mean)
v_res <- apply(dat_res, 2, var)
dat_res_std =cbind((dat_res[,1]-m_res[1])/sqrt(v_res[1]),(dat_res[,2]-
m_res[2])/sqrt(v_res[2]),(dat_res[,3]-m_res[3])/sqrt(v_res[3]))

data1 = cbind(fit1, fit2)
data2 = cbind(fit2, fit3)

#######################
# Copula Fit Oil vs SP500   #
#######################
fnorm1 =
fitCopula(copula=normalCopula(omega1,dim=2),data=data1,method="ml")
fnorm1.rho = coef(fnorm1)[1]
persp(normalCopula(dim=2,fnorm1.rho),dCopula)


ftcop1 = fitCopula(copula=tCopula(omega1,dim=2),data=data1,method="ml")
ftcop1.rho = coef(ftcop1)[1]
ftcop1.df = coef(ftcop1)[2]
persp(tCopula(dim=2,ftcop1.rho, df = ftcop1.df),dCopula)

fgumbel1 = fitCopula(copula = gumbelCopula(2, dim=2), data = data1, method =
"ml")
fgumbel1.par= coef(fgumbel1)[1]
persp(gumbelCopula(dim=2,fgumbel1.par),dCopula)

ffrank1 = fitCopula(copula = frankCopula(cor_tau1, dim = 2), data = data1,
method = "ml")
ffrank1.par = coef(ffrank1)[1]
persp(frankCopula(dim=2,ffrank1.par),dCopula)

fclayton1 = fitCopula(copula = claytonCopula(cor_tau1, dim=2), data = data1,
method = "ml")
```

```
fclayton1.par = coef(fclayton1)[1]
persp(claytonCopula(dim=2,fclayton1.par),dCopula)

fjoe1 = fitCopula(copula=joeCopula(2,dim=2),data=data1,method="ml")
fjoe1.par = coef(fjoe1)[1]
persp(joeCopula(dim=2,fjoe1.par),dCopula)

par(mfrow=c(2,3))
persp(normalCopula(dim=2,fnorm1.rho),dCopula,main="Gaussian")
persp(tCopula(dim=2,ftcop1.rho, df = ftcop2.df),dCopula,main="student-t")
persp(gumbelCopula(dim=2,fgumbel1.par),dCopula, main="Gumbel")
persp(claytonCopula(dim=2,fclayton1.par),dCopula,main="Clayton")
persp(frankCopula(dim=2,ffrank1.par),dCopula, main="Frank")
persp(joeCopula(dim=2,fjoe1.par),dCopula,main="Joe")
par(mfrow=c(1,1))

#########################
# Copula Fit Oil vs NASDAQ #
#########################

fnorm2 =
fitCopula(copula=normalCopula(cor_tau2,dim=2),data=data2,method="ml")
fnorm2.rho = coef(fnorm2)[1]
persp(normalCopula(dim=2,fnorm2.rho),dCopula)


ftcop2 = fitCopula(copula=tCopula(cor_tau2,dim=2),data=data2,method="ml")
ftcop2.rho = coef(ftcop2)[1]
ftcop2.df = coef(ftcop2)[2]
persp(tCopula(dim=2,ftcop2.rho, df = ftcop2.df),dCopula)

fgumbel2 = fitCopula(copula = gumbelCopula(2, dim=2), data = data2, method =
"ml")
fgumbel2.par= coef(fgumbel2)[1]
persp(gumbelCopula(dim=2,fgumbel2.par),dCopula)

ffrank2 = fitCopula(copula = frankCopula(cor_tau2, dim = 2), data = data2,
method = "ml")
ffrank2.par = coef(ffrank2)[1]
persp(frankCopula(dim=2,ffrank2.par),dCopula)

fclayton2 = fitCopula(copula = claytonCopula(cor_tau2, dim=2), data = data2,
method = "ml")
fclayton2.par = coef(fclayton2)[1]
```

```
persp(claytonCopula(dim=2,fclayton2.par),dCopula)

fjoe2 = fitCopula(copula=joeCopula(2,dim=2),data=data2,method="ml")
fjoe2.par = coef(fjoe2)[1]
persp(joeCopula(dim=2,fjoe2.par),dCopula)

par(mfrow=c(2,3))
persp(normalCopula(dim=2,fnorm2.rho),dCopula,main="Gaussian")
persp(tCopula(dim=2,ftcop2.rho, df = ftcop2.df),dCopula,main="student-t")
persp(gumbelCopula(dim=2,fgumbel2.par),dCopula, main="Gumbel")
persp(claytonCopula(dim=2,fclayton2.par),dCopula,main="Clayton")
persp(frankCopula(dim=2,ffrank2.par),dCopula, main="Frank")
persp(joeCopula(dim=2,fjoe2.par),dCopula,main="Joe")
par(mfrow=c(1,1))

############################
# Copula estimate Oil vs SP500  #
############################

fnorm1.u = rCopula(521,normalCopula(dim=2,fnorm1.rho))
taplot(fnorm1.u[,1],fnorm1.u[,2],pch='.',col='blue')
cor(fnorm1.u,method='kendall')

ftcop1.u = rCopula(521,tCopula(dim=2,ftcop1.rho,df=4))
plot(ftcop1.u[,1],ftcop1.u[,2],pch='.',col='blue')
cor(ftcop1.u,method='kendall')

fgumbel1.u = rCopula(521,gumbelCopula(dim=2,fgumbel1.par))
plot(fgumbel1.u[,1],fgumbel1.u[,2],pch='.',col='blue')
cor(fgumbel1.u,method='kendall')

ffrank1.u = rCopula(521,frankCopula(dim=2,ffrank1.par))
plot(ffrank1.u[,1],ffrank1.u[,2],pch='.',col='blue')
cor(ffrank1.u,method='kendall')

fclayton1.u = rCopula(521,claytonCopula(dim=2,fclayton1.par))
plot(fclayton1.u[,1],fclayton1.u[,2],pch='.',col='blue')
cor(fclayton1.u,method='kendall')

fjoe1.u = rCopula(521,joeCopula(dim=2,fjoe1.par))
plot(fjoe1.u[,1],fjoe1.u[,2],pch='.',col='blue')
cor(fjoe1.u,method='kendall')
```

```
##############################
# Copula estimate Oil vs NASDAQ #
##############################

fnorm2.u = rCopula(521,normalCopula(dim=2,fnorm2.rho))
plot(fnorm2.u[,1],fnorm2.u[,2],pch='.',col='blue')
cor(fnorm2.u,method='kendall')

plot(Oil.ret,SP500.ret ,main='Returns')
points(fnorm2.u[,1],fnorm2.u[,2],col='red')
legend('bottomright',c('Observed','Simulated'),col=c('black','red'),pch=21)

ftcop2.u = rCopula(521,tCopula(dim=2,ftcop2.rho,df=4))
plot(ftcop2.u[,1],ftcop2.u[,2],pch='.',col='blue')
cor(ftcop2.u,method='kendall')

fgumbel2.u = rCopula(521,gumbelCopula(dim=2,fgumbel2.par))
plot(fgumbel2.u[,1],fgumbel2.u[,2],pch='.',col='blue')
cor(fgumbel2.u,method='kendall')

ffrank2.u = rCopula(521,frankCopula(dim=2,ffrank2.par))
plot(ffrank2.u[,1],ffrank2.u[,2],pch='.',col='blue')
cor(ffrank2.u,method='kendall')

fclayton2.u = rCopula(521,claytonCopula(dim=2,fclayton2.par))
plot(fclayton2.u[,1],fclayton2.u[,2],pch='.',col='blue')
cor(fclayton2.u,method='kendall')

fjoe2.u = rCopula(521,joeCopula(dim=2,fjoe2.par))
plot(fjoe2.u[,1],fjoe2.u[,2],pch='.',col='blue')
cor(fjoe2.u,method='kendall')

#####################################
# Cramer Von Mises Statistic Oil vs SP500 #
#####################################
itau1 = cor_tau1[1]

fnorm1.d = gofCopula(normalCopula(fnorm1.rho), Oil.SP500.ret.t)

ftcop1.d = gofCopula(tCopula(itau1, df.fixed=TRUE),
Oil.SP500.ret.t,simulation="mult")

thG1 = iTau(gumbelCopula(), itau1)
fgumbel1.d = gofCopula(gumbelCopula(thG1), Oil.SP500.ret.t)
```

```
thC1 <- iTau(claytonCopula(), itau1)
fclayton1.d = gofCopula(claytonCopula(thC1), Oil.SP500.ret.t)

ffrank1.d = gofCopula(frankCopula(itau1), Oil.SP500.ret.t)

thJ1 = iTau(joeCopula(), 0.5)
fjoe1.d = gofCopula(joeCopula(thJ1), Oil.SP500.ret.t)

fnorm1.d
ftcop1.d
fgumbel1.d
fclayton1.d
ffrank1.d
fjoe1.d

#########################################
# Cramer Von Mises Statistic Oil vs NASDAQ  #
#########################################
itau2 = cor_tau2[1]

fnorm2.d = gofCopula(normalCopula(fnorm2.rho), Oil.NASDAQ.ret.t)

ftcop2.d = gofCopula(tCopula(itau2, df.fixed=TRUE),
Oil.NASDAQ.ret.t,simulation="mult")

thG2 = iTau(gumbelCopula(), itau2)
fgumbel2.d = gofCopula(gumbelCopula(thG2), Oil.NASDAQ.ret.t)

thC2 <- iTau(claytonCopula(), itau2)
fclayton2.d = gofCopula(claytonCopula(thC2), Oil.NASDAQ.ret.t)

ffrank2.d = gofCopula(frankCopula(itau2), Oil.NASDAQ.ret.t)

thJ2 = iTau(joeCopula(), 0.5)
fjoe2.d = gofCopula(joeCopula(thJ2), Oil.NASDAQ.ret.t)

fnorm2.d
ftcop2.d
fgumbel2.d
fclayton2.d
ffrank2.d
fjoe2.d
```